Are Universities Fair? Risking the Endowment for Future Generations*

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Abstract

University endowments invest more than 70% of their assets in risky securities. This large allocation to risky assets matches the investment advice provided by standard consumption and portfolio choice models with long-horizon individual investors. But universities are not individuals. In this paper, we analyze the universities' infinite-horizon portfolio choice problem under the paradigm of maintaining intergenerational fairness with a zero rate of time preference. First, we quantify the oft-cited mandate of university trustees to maintain a policy of fairness across generations without any favoritism for the present over the future. Second, we show that the fairer the universities, the less they invest in the risky asset. In the limit, they forego the risky asset completely to provide constant consumption. Third, we explore how if generations are allowed to have differing risk aversions or if universities neglect extreme left-tail return events, then the contradiction between intergenerational fairness and risky asset allocation can be lessened.

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1 Introduction

Universities are perpetual ivory towers where knowledge is created via research and disseminated via teaching. But rather than using their alumni donations to build higher towers and shine the light of learning more widely, university endowment offices invest more than 70% of their assets in risky external projects, e.g., equities, hedge funds, real estate, and commodities. This choice of asset allocation is consistent with the advice provided by standard models of consumption and portfolio choice for individual long-term investors Campbell and Viceira (2002), but should it be? Universities are not simply individuals with a long horizon and a different objective function must be defined given their unique characteristics.

In a seminal paper on permanent endowment income, Tobin (1974) unambiguously states the objective function of a university:

> "The trustees of an endowed institution are the guardians of the future against the claims of the present. Their task is to *preserve equity among generations*. The trustees of a university like my own assume the institution to be *immortal*. They want to know, therefore, *the rate of consumption from endowment which can be sustained indefinitely*. [...] In formal terms, the trustees are supposed to have a *zero subjective rate of time preference*." [Emphasis added]

In this paper, we quantify the objective of fairness, also known as equity, from the point of view of a university. This allows us to model the optimal consumption and portfolio choice of a university concerned with intergenerational fairness. Under reasonable conditions, the model delivers optimal portfolio allocations substantially different from those empirically observed. This difference leaves us with a puzzle: Why do university endowments invest so much in risky assets?

Intuitively if the university is to provide its current members with more consumption than can be supported from the endowment at the risk-free rate without surely depleting its endowment, then either its future members must receive less consumption than today's or the university must take on risk. The first choice is clearly intergenerationally unfair. The second choice supporting higher current consumption by investing the endowment in risky assets to earn the equity premium means the future would not need to *expect* reduced consumption, seemingly solving the problem. However, unfairness remains: the current generation has taken for itself a large piece of the pie and handed the future generation a gamble.

The key to making this intuition precise requires us to take a stand on the university's objective function. In doing so, we extend the previous literature by quantifying the notion of intergenerational fairness previously left imprecise. In modeling the university as a *fair consumer*, we define fairness by giving the university a concave meta-utility over each generation's individual utility function. The concavity of this meta-utility function measures how much the university gains by giving one generation more at the expense of giving another generation less, even probabilistically less.

By increasing the concavity of the university's meta-utility, we analyze the impact of increased fairness on the university's allocation to the risky asset and consumption policy also known as the endowment payout policy. We show that the fairer the university, the lower its allocation to the risky asset. In the limit, the university completely abstains from investing in the risky asset leading to a constant stream of consumption. As a result, we find the fair consumer paradigm fails to explain why universities hold so much of their endowment in risky assets.

One of the most common arguments in favor of an asset allocation heavily tilted towards risky assets, stocks in particular, is that stocks are less risky than bonds in the long run. Siegel (2008) makes this claim in a convincing manner using our sample of 100 years of U.S. returns. Given a university's extremely long-term horizon, it is tempting to further argue that they are uniquely placed to ride out the bad times and take advantage of long-term mispricings. There are two problems with using Siegel's argument as a justification for university endowments to bear generally high levels of risk. First, Siegel's argument about safety in the long-run has not been extended to hedge funds, real estate, and commodities, which form a substantial portion of the average endowment's asset allocation. Second, as Black (1976) points out, having a long-term horizon is not a license to take risk.

Imagine that you have just read that an anti-aging drug was just discovered. Would you drive home faster or slower tonight? Potential immortality and the long horizon it implies does not guarantee a higher risk-bearing capacity. Universities have the potential to live forever, but risking the endowment for future generations means that there may not be future generations. Public pension plans face a similar dilemma and heavily tilting their asset allocation towards riskier assets to solve under-funding problems can lead to lower benefits for future generations when bad returns do occur. Both of these trade-offs can be moderated by preferences for risk aversion and fairness, neither of which necessarily fall with an increasing horizon.

Another common argument in favor of investing university endowments in risky assets is inflation hedging. An endowed gift is supposed to provide a constant stream of *real* cash flows. An investment in nominal riskless bonds may lose value in real terms over time if realized inflation is sufficiently high. However, this does not imply that universities can mitigate this erosion in real value by shifting their holdings towards risky assets. This shift would only be beneficial if risky asset returns were positively correlated with inflation – something that is empirically untrue (see Fama and Schwert (1977), Bodie (1976), and Fama (1981), among many others).

Two assets have a positive correlation structure with inflation. First are the recently introduced Treasury Inflation Protected Securities (TIPS). Campbell and Viceira (2002) make a strong case for the fact that TIPS are the risk-free asset for a long-term investor and this should apply to universities as well. However, the average allocation to TIPS by endowments has remained small for a variety of reasons. Second is local real estate which provides protection for the university against a more limited form of inflation, namely the cost of land for expansion (Merton, 1993). Nevertheless, neither example provides broad license to invest in real estate and commodities, among other risky assets, as a means to hedge inflation.

Our notion of fairness builds on that of Rawls (1971), who argues for the max-min criterion as an implementation of justice as fairness. However, our definition provides more flexibility than that of Rawls', allowing us to model all ranges of fairness – from complete indifference between the present and the future to complete fairness as exemplified by the max-min criterion.[1] Embedded in our fairness definition is an expectation over the utility of all potential future university members, which is in the spirit of Rawls' veil of ignorance. Implementing fairness via an expectation over future university membership allows us to have a zero rate of time preference while overcoming the technical difficulties of divergent infinite sums that typically accompany intertemporal optimization problems without discounting.

In current models of portfolio choice, there only exists closed-form analytical solutions for a restricted set of objective functions and asset-return specifications. In order to capture more realistically the optimization problem faced by universities, we move outside this class of objective functions and solve our model numerically via a state space discretization that approximates the infinite horizon solution.

Universities and their endowments have a long history of investing in risky external projects. In the 19th century, Lafayette and Marietta Colleges attempted to grow their endowment by investing in mulberries and silkworms, both of which turned out to be extremely poor choices. More recently, the University of Rochester lost a large fraction of its endowment because of a non-diversified investment in Kodak – a striking case of home bias. This risk-taking behavior in external markets, as opposed to internal risk-taking by investing in new professors and new lines of research, is precisely what we try to explain in this paper.

Since we find that the fairness objective function cannot explain the observed investment strategies of universities, in a companion paper (Gilbert and Hrdlicka, 2011) we model an alternative objective of universities. In that paper we focus on universities as producers of social goods, research and teaching, and we model the trustees' mandate of investing an endowment that must fund internal projects. Under this mandate, trustees maximizing future cash flows face the trade-off between investing externally or internally.

[1] See Section 3 for definitions and explanations of fairness and the max-min criterion.

External investment occurs via endowment funds in the capital markets. Internal investment consists of funding current research and teaching, which will generate knowledge and future donations. While facing this trade-off, trustees must simultaneously balance the demands of a diverse set of constituents: internal stakeholders such as faculty and students versus external stakeholders such as donors, alumni, and society at large.

2 Literature Review

The theoretical literature on optimal investment and portfolio choices with a focus on universities is small. The main reason for this is the lack of definition of the university's objective function. However, the empirical literature on university endowments has grown in recent years, probably due to the extraordinary returns generated by Harvard and Yale. We review both sides as they pertain to our research question.

2.1 Theoretical Literature

The chief investment officer of the Yale University endowment, David Swensen (2009), states that universities benefit from an endowment as it helps maintain an independent and stable source of revenue, thereby guaranteeing the institution's intellectual freedom and enhancing educational excellence. In line with these goals, he writes that endowment managers have two primary, and conflicting, goals: provide stable (predictable) cash flows to the operating budget of the university as well as maintain the purchasing power of the endowed gifts. His reasoning is heavily based on Tobin (1974) who develops a basic formula for permanent endowment payout, which itself relies on the notion of intergenerational fairness, or equity.

Swensen argues as others before him that because stocks are apparently safer in the long run, universities should invest heavily in risky assets. Tobin takes no such stand on the level of risk universities should bear. Similarly, Merton (1993) takes as given the level of risk the university tolerates when he maps his earlier work on optimal portfolio choice in continuous-time onto a university endowment framework where consumption is defined as expenditure on internal projects. Merton's conclusions on asset allocation are unchanged in the sense that the endowment should hedge against adverse changes in the investment opportunity set.

Black (1976) denounces as ungrounded the typical argument that because of their very long horizon, universities can afford to take more risk. Since risk must eventually be borne by individuals, such risky asset allocation unfairly punishes a generation at the expense of another. Hansmann (1990) expresses a skeptical viewpoint on the reasons for the existence of endowments. He argues that, beyond motives of creating a buffer against bad times and thereby insuring the long-run survival and intellectually independence of universities, all other arguments for the existence of endowments are unpersuasive.

Constantinides (1993) discusses the importance of choosing the correct objective function for a university. He gives two stylized examples of different objective functions leading to dramatically different investment policies.

2.2 Empirical Literature

The empirical literature on university operations and governance is small (Winston (1999) and Ehrenberg (1999)) and mainly highlights the differences between higher education and other non-profits.

Lerner, Schoar, and Wang (2008) report that universities with large endowments and those with high SAT scores enjoy consistently higher returns than universities with smaller endowments and lower SAT scores. The former universities also invested earlier and more in so-called alternative strategies (hedge funds, etc.) compared to the latter universities.

Brown et al. (2010) investigate the effects of endowment shocks on university operations. They find that, following negative shocks, universities consume less out their endowments than specified by their payout rule, whereas they do not significantly deviate from the rule following positive shocks.[2] To make up for the reduced endowment consump-

[2] Most universities employ a payout rule in which they consume a fixed percentage (4-7%) of the previous

tion, universities scale down fundamental parts of their operations and cut perquisites to students, faculty, and staff.

Dimmock (2010) presents evidence that universities whose non-financial income (tuition, fees, grants, etc.) is more volatile invest more of their endowment in fixed income assets and reduce their allocation to risky assets. Brown, Garlappi, and Tiu (2010) report evidence of skill in security selection on the part of university endowment managers.

3 Intergenerational Fairness

Fairness, or equity, is a feature of people's preferences just as is the desire for more consumption and the avoidance of risk. Economists have found evidence of the preference for fairness in experimental studies of adults (Fehr and Schmidt, 2006). Experiments have also revealed that the preference for fairness begins between the ages of 5 and 6 (Fehr et al., 2008).[3]

Rawls (1971) uses this preference for fairness as foundation of a just society. Fairness is central to the discussion of many society level problems beyond justice: scarce resource allocation, exhaustible resource allocation, and society's level of saving. More recently fairness has be used to address questions regarding social security, global warming, observed distribution of and persistence of wealth within and across families.[4]

Tobin (1974) brings fairness to the analysis of university endowments. He states the university's objective function as one of maintaining equity across generations from today to the infinite future. Part of this equity requires a zero rate of time preference. Based on this objective, Tobin leaps to a goal of maximizing sustainable consumption from a given investment with predetermined level of risk, and he finds that only permanent capital gains should be consumed, while transitory capital gains should be ignored. Tobin's definition of

few years' average market value of the endowment.

[3]There is an extensive literature in microeconomics and game theory on fairness (Kahneman et al. (1986), Rabin (1993), Fehr and Schmidt (1999), and Fehr and Gächter (2000), to cite just a few), but we depart from these papers by focusing on intertemporal choices.

[4]See Hartwick (1977), Ramsey (1928), Rawls (1971), Arrow (1973), Becker and Tomes (1979), Gordon and Varian (1988), Gollier (2008), Partha Dasgupta and Barret (1999).

fairness does not allow him to address the level of risk universities should undertake in their endowments nor consider the implication of stochastic returns beyond the decomposition of capital gain types.

In this paper, we take as given the qualitative notion of fairness as the objective of a university. We add to the literature with a precise mathematical formulation of the preference for fairness and with a method to address a zero rate of time preference in a tractable way. These innovations allow us to explicitly analyze the level of risk a university bears in its endowment and the payout policy such an investment strategy can support.

Model of Fairness. Rawls (1971) advocates for the max-min criterion of fairness across generations labeled by i:

$$\max_{c_i} [\min\{u(c_1), u(c_2), \ldots\}] \tag{1}$$

which summarizes the concept that one generation cannot be made happier by increasing its consumption unless all other generations also increase their consumption. Leontief indifference curves are the representation of the max-min criterion. Rawls further argues that society should consider the fate of the lowest individual, i.e., the max-min criterion within a generation. Rawls acknowledges that lesser degrees of fairness from the max-min might be necessary but does not specify a form for these intermediate levels of fairness.

We model fairness as a meta-utility function over the utility of individual members of the university within a generation and across time. Fairness inherently involves comparisons across individuals and the meta-utility function represents the university's preference for tolerating inequality across individuals and generations. The more concave the meta-utility function the less the university is willing to tolerate giving one generation more at the cost of giving another less.

Each generation has standard CRRA utility over consumption labeled as $u(c_t)$ and the university's fairness utility over a single generation's utility is given by $V[u(c_t)]$:

$$u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma} \qquad \text{and} \qquad V\left[u(c_t)\right] = \frac{\left(-\frac{1}{u(c_t)}\right)^{1-f}}{1-f} \tag{2}$$

where γ is the coefficient of risk aversion of f is the fairness coefficient.

Our fairness parameter f allows us to model a range of preferences for fairness. What we term fairness neutrality is achieved when $f = 2$. In this case the university's utility function collapses to standard CRRA. As f increase so does the university's preference for fairness. As f goes to infinity this meta-utility approaches Leontief preferences of the max-min criterion advocated by Rawls.[5] Figure 1 shows a sampling of the indifference curves for a two-period (two-generation) model as f increases toward infinity.

Basis of Fairness Model. We incorporate the comparison across generations and the zero rate of time preference via an expectation in the spirit of Rawls' *veil of ignorance*: The principle that fairness requires individuals to consider the decisions they would make if all distinguishing facts about themselves, such as gender, race, skill and importantly in our context, the generation in which one is born, were unknown at the decision time.[6] The university's total utility function as of date t_0 is

$$E_{t_0}\left[V[u(c_t)]\right] \tag{3}$$

This expectation is over the standard state uncertainty *and* over the future generation of the university that the hypothetical decision maker may randomly show up as. This second component of the expectation follows directly from Rawls' veil of ignorance. The result of such a reduced information decision should lead to fair policies, that in our case, any new incoming freshman student is satisfied with, no matter when she is born. In equilibrium,

[5] The lesser degrees of fairness are critical when considering intergenerational problems. Rawls abandoned the max-min criterion for such problems because it leads to no savings whatsoever, but in doing so jumped to the other extreme of fairness neutral. Our model encapsulates all degrees of fairness in a parsimonious way.

[6] This is also called the *original position*.

adjusting for growth in the economy and hence in the university's scope of operations, she is not envious of other generations, past or future.

To make clear the distinction between these two sources of uncertainty, we can use iterated expectations to rewrite this utility function as

$$E_{t_0}\Big[E\big[V[u(c_t)]|t\big]\Big] \tag{4}$$

where the outer expectation is over only the generational uncertainty and the inner expectation is over the standard state uncertainty. The expectation over generation uncertainty is a sum against the population density across and within generations. If we assume constant generations sizes this reduces to a uniform distribution. Under the uniform distribution we can rewrite the university's utility function as

$$E_{t_0}\left[\lim_{T\to\infty}\sum_{i=0}^{T}\frac{1}{T}V[u(c_{t_0+i})]\right]$$

Provided that the rate of population growth and rate consumption growth combined are not too great this sum will converge. This specification of the utility function allows for a zero rate of time preference without the standard problem of a divergent sum.[7]

Though our functional form for fairness resembles risk aversion, they are clearly different concepts. Fairness preferences exist even in a determinist world (see Arrow (1973) and Solow (1974)), and hence can be added to risk aversion when a world is also subject to uncertainty. Fairness allows us to consider responses to different personal characteristics including risk aversion that changes across individuals. We explore this notion that each generation may have a different level of risk aversion later in the paper.

[7] In the standard infinite horizon model with the time discount factor set equal to one, $\delta = 1$, representing a zero rate of time preference, the standard additive utility framework: $\sum_{t=0}^{\infty}\delta^t u(c_t)$ leads to a divergent sum unless consumption levels collapse at a sufficiently high rate.
Our innovation contributes to the literature on the convergence of infinite horizon utility functions and the necessity to have positive personal or social discount rates that began with Tinbergen (1956) and Chakravarty (1962). Our innovation is also related to the literature on what types of preferences without time preference and what assumptions are necessary define preferences over infinite horizons without time preference (see Koopmans (1960), Diamond (1965), Svensson (1980), and Basu and Mitra (2003)).

Despite being different aspect of preferences, in our models fairness leads to behavior similar to increased risk aversion. A higher fairness preference pushes investment away from risky assets. This application of risk aversion resembles the results of the robustness literature (see Hansen and Sargent (2007)) that another feature of preference can lead to behavior similar to increased risk aversion.

Related Literature. Solow (1974) critiques the max-min criterion for intergenerational equity for similar reasons as Rawls and highlighted the perpetually low levels of consumption. Arrow (1973) modifies Rawls' max-min criterion to consider a generation which has preferences over its own consumption and that of the following generation.[8] This modification allows savings to occur. Arrow also considers lengthening the horizon of future consumption a current generation gains utility over. He finds that when the current generation has a preference over consumption with an infinite horizon, this modified max-min criterion is equivalent to that of Rawls applied intergenerationally.

Becker and Tomes (1979) apply Arrow's modified Rawlsian fairness at the family intergenerational level. They use fairness within the family to explain the observed society-wide distribution of income, wealth and consumption. They also address the persistence level of these variables. In a discussion of global warming, Partha Dasgupta and Barret (1999) also use a modified version of Rawls' definition of fairness. This modification is the one closest to ours.

The appropriate rate of time preference is tied to any discussion of fairness in an intertemporal setting. Ramsey (1928) argues forcefully for no time preference. Rawls' veil of ignorance precludes a rate of time preference, for behind the veil of ignorance you do not know what time period you will live in, and hence you consider all periods equally. As highlighted in the introductory quote, Tobin (1974) also argues for no time preference.

A zero rate of time preference does not mean that one should have a zero discount rate when choosing between alternative investment projects at the individual or societal level.

[8] More precisely: $\max\left[\min_t W(c_t, c_{t+1})\right]$ where $W(c_t, c_{t+1}) = U(c_t) + \beta U(c_{t+1})$.

Discount rates can come purely from the technology available for example (Cochrane, 1991). Partha Dasgupta and Barret (1999), in addition to discussing a range of social discounting methods, derive social time preference from underlying economic fundamentals such as production ability.

4 Fair Consumption and Portfolio Choice

In this section, we analyze the university's asset allocation decision when its objective function is to *fairly* maximize the sum of every generation's expected utility over consumption. Wealth is accumulated by investing in the endowment, whose asset allocation between the risky and risk-free assets is endogenously chosen. Consumption from the endowment is also optimally chosen each period and allows the university to operate and accomplish its stated goals of education and research. Using the definitions of fairness developed in the previous section, we solve an infinite horizon model of consumption and investment where, as fairness increases, the endowment's allocation to the risky asset decreases dramatically, to zero in the limit of absolute fairness.

4.1 Model Setup

The university is modeled as an agent who uses all relevant information to make optimal consumption and portfolio allocation decisions in every period t.[9] At each period, the university is populated by a generation of faculty, staff, and students, who only live for one period. The university's wealth is invested in the endowment, from which each generation's consumption must be drawn. The remaining wealth at each time is allocated between a risk-free asset with constant return and a risky asset with stochastic return. The intertemporal budget constraint therefore is

$$\tilde{W}_{t+1} = (W_t - c_t) \times (\alpha_t \tilde{R}_{r,t+1} + (1 - \alpha_t) R_f) \tag{5}$$

[9]Our model is closely related to Campbell and Viceira (1999) with the main difference being their use of Epstein-Zin utility and our use of fairness utility.

where W_t is wealth at time t, c_t is consumption at time t, α_t is the portfolio weight invested in the risky asset at time t, $\tilde{R}_{r,t+1}$ is the stochastic gross return on the risky asset at time $t+1$, and R_f is the constant gross risk-free rate of return. W_t basically is the value of the endowment at time t, from which consumption occurs and whose asset allocation is chosen endogenously at each time period.

As explained in the previous section, each generation has constant relative risk aversion (CRRA) utility $u(c_t)$ over its consumption c_t and the university has a fair concave meta-utility $V[u(c_t)]$ over each generation's utility:

$$u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma} \qquad \text{and} \qquad V\left[u(c_t)\right] = \frac{\left(-\frac{1}{u(c_t)}\right)^{1-f}}{1-f} \tag{6}$$

where γ is the coefficient of risk aversion of f is the fairness coefficient. We assume that all generations have the same coefficient of relative risk aversion. In an extension of the model, we relax this assumption by allowing each generation to have a different level of risk aversion.

The university aims to be immortal and hence has an infinite investment horizon. The university solves

$$\max_{c_t,\alpha_t} E_t \lim_{T\to\infty} \sum_{i=0}^{T} \frac{1}{T} V\left[u(c_{t+i})\right]. \tag{7}$$

While this model does not consider either the productive nature of universities or the characteristics of their stakeholders, something we analyze in a companion paper (Gilbert and Hrdlicka, 2011), it does capture several realistic elements of universities. First, endowments do typically represent a significant fraction of universities' liquid wealth. Second, the annual payout from the endowment maps straightforwardly into the ratio of consumption to wealth at each period. Third, anecdotal evidence shows that most university provosts view their various divisions and departments as *cost centers* to which they have to allocate funds, and which will hopefully generate intangible items such as research and teaching. In this sense, viewing university generations (faculty, staff, and students) as consumers of resources is reasonable. Fourth, the model's implicit aim of smoothing consumption over time is consistent with the trustees' mandate of providing a predictable stream of payout from the endowment.

The above form of the utility function does not allow us to obtain closed-form analytical solutions for the optimal consumption and investment policies. We therefore resort to numerical solutions by state-space discretization. The methodology is explained in details in Appendix B.

4.2 Traded Assets

The university's endowment has two assets to choose from when determining its optimal asset allocation each period: a risk-free asset with constant return and a risky asset with stochastic return. No short positions in either asset is permitted. The assumption of constant return for the risk-free asset follows Campbell and Viceira (1999) and is made mainly for tractability reasons. While we acknowledge that endowments do invest across the entire risky term structure of interest rates, adding state variables for each predictable component of the yield curve is analytically and numerically unfeasible.

An important assumption of the model is the dynamics of returns for the risky asset. It is a well-known fact that, if returns are i.i.d., then a CRRA investor will behave myopically in her investment decision. Since the investment opportunity set is unchanged over time, the consumption decision does not depend on the distribution of returns and the asset allocation decision only depends on the contemporaneous distribution of returns. The CRRA investor facing i.i.d. returns behaves like a one-period investor and the investment horizon plays no role in the analysis.

Aside from the government and public pension plans, universities arguably have the longest investment horizon of all institutional investors. To take into account the non-myopic behavior of university endowments, we model returns based on Campbell and Viceira (1999), which is explained in details in Appendix A. We assume that expected excess stock returns follow a highly persistent process, making returns mean-reverting: if expected returns are high today then returns are likely to be low tomorrow. Using this model, Campbell and Viceira (1999) show that a long-term investor will behave more aggressively than a one-period investor and will therefore invest more in the risky asset. They also show that a buy-and-hold strategy suffers significant welfare losses from not timing the market and taking advantage of the predictability in expected returns.

4.3 Asset Allocation Results

Panel A of Figure 2 shows how the optimal allocation to the risky asset (α_t) changes as the investment horizon increases from $T = 1$ to 100 periods (years). When returns are i.i.d. (dashed line), the optimal portfolio choice is independent of the horizon, as expected. The top line has a fairness coefficient of 2, which corresponds to the basic CRRA case of Campbell and Viceira (1999). We see that, as the investment horizon lengthens, the investor allocates an increasing amount to the risky asset. In addition, all optimal allocations are greater than in the i.i.d. case, since expected returns are now predictable. The other curves, from top to bottom, are for fairness coefficients of 3, 4, 5, and 10. The fairer the university wants to be, the less it allocates to the risky asset in equilibrium. Even though the amount increases as the horizon lengthens, both the level and the change in the optimal allocation significantly decreases as fairness increases.

Universities aim to be immortal, so we make sure our analysis holds for extremely long horizons. In Panel B of Figure 2, we show the same analysis as in Panel A, but for an investment horizon of 1,000 years. Beyond a 100-year horizon, the optimal asset allocation is almost constant and the *fair* asset allocation remains significantly lower than the standard CRRA asset allocation.

Figure 3 shows the optimal allocation to the risky asset versus the university's coefficient of fairness for different investment horizons: one period (i.i.d. returns), 25, 50, 75, and 100 years, respectively from bottom to top. As fairness increases, the optimal risky weight decreases dramatically, from around 60% to about 10%. Per the results of Figure 2, the longer the horizon, the higher the allocation to the risky asset. However, as fairness increases, the optimal allocation falls for all horizons.

The standard investment advice that long-horizon investors should hold a significant portion of their portfolio in the risky asset can be easily seen on Figure 3. Indeed, for a fairness coefficient of two, the model collapses to the standard CRRA model of consumption and portfolio choice of Campbell and Viceira (1999). So at a fairness coefficient of two, with non-iid returns, and an investment horizon of 100 years, the optimal asset allocation is 85% in the risky stock. However, this standard investment advice does not take into account one unique characteristic of universities, namely that they aim to be intergenerationally fair.

Figure 4, which is partly based on Figure 4.1 in Campbell and Viceira (2002), also shows how misleading the standard investment advice for individual long-term investors can be when applied to universities. The figure shows the optimal risky asset weight as a function of the risky asset's expected return. The dashed horizontal line highlights the myopic buy-and-hold strategy of an investor who assumes that the conditional expected excess stock return is equal to the unconditional mean (5% on this figure). The light solid line (tactical allocation) shows the optimal holding for a one-period investor who observes the conditional mean. The steepest solid line (strategic allocation) shows the standard advice that a long-horizon investor will hold more of the risky asset than a one-period investor. In comparison, the flattest sold line shows the optimal policy for a university with long-horizon and a fairness coefficient of five: its risky stock holding falls dramatically as it aims to ensure a constant consumption stream.

The impact of the change from standard δ discounting to the equal-weighting of all future time periods does not play a huge role in the analysis. Figure 5 shows the optimal allocation to the risky asset as a function of the horizon when fairness is 10 for both the δ and equal-weighting methods. Even at very long horizons, the difference in optimal risky asset weight is less than 2%.

4.4 Consumption Results

In Figure 6, we plot the standard deviation of consumption as a function of the investment horizon, for different levels of fairness. From top to bottom, fairness increases from 2 to 15. The longer the investment horizon, the more volatile expected consumption is. However, as fairness is increased, the volatility of consumption falling nearly to zero when fairness is 15.

In Table 1 we present statistics about the university consumption policy for fairness levels 2, 5, 10 and 15 under the zero time preference setting and for fairness level 10 under the standard discounting of the future. We see that the university's payout ratio out of the endowment falls from 2% to 0.5% as fairness increases. We also see that the payout ratio with standard time discounting is higher at 0.85% than the 0.62% payout ratio for the same fairness level under a zero rate of time preference. As fairness increases we also see the expected endowment return fall from 4.37% to 0.83% as the university invests less in the risky asset. There is little difference in the expected endowment return as the time preference is changed between the cases we consider.

Though both the payout ratio and expected endowment return fall as fairness rises, the ratio of the two rise. This ratio represents the expected fraction of the endowment return paid out each period. Under a zero rate of time preference for a fairness of two the ratio is 47.51% gradually rising to 60.96% for a fairness level of 15. This rise makes sense for in the limit of fairness the university invests only in the risk free asset and pays out its entire return each period. When there is a positive rate of time preference this ratio is higher at 79.50%. This makes sense for the university values consumption today and hence saves less of today's returns for the future.

To show the impact of unexpected returns we tabulate the impulse response function for a one standard deviation positive shock to the risky asset return. We normalize the response by the consumption level at the mean return of the risky asset. We see that there is 13.31% increase after such a shock for a fairness level of 2. This response falls to 1.84% for a fairness level of 15. The fall is due to a double dampening effect: a fairer university invests less in the risky asset limiting its exposure to the shock and a fairer university has a lower payout rule which provides more smoothing. There is little difference between the shock response for a university with a zero rate of time preference versus one with a positive rate of time preference.

From our analysis of universities as *fair consumers*, we conclude that this paradigm fails to answer our research question. If universities' objective function is to be intergenerationally equitable, then their endowment should hold very little of the risky asset. The endowment should be primarily a precautionary saving vehicle aimed at ensuring stable and constant consumption across all generations.

5 Model Extensions

5.1 Random Risk Aversion

We now consider an extension to the model that allows university members (generations) to have heterogeneous characteristics beyond simply the period in which they live. The key characteristic we allow to vary across generations is risk aversion. By assuming that, at each time period, the university faces a new generation whose risk aversion is drawn from a particular distribution, we are able to analyze how this randomness interacts with the university's preference for fairness. More precisely, the fairness preference leads to a non-market aggregation mechanism of heterogeneous risk aversion, which impacts asset holdings and consumption choices.

A large body of literature discusses the heterogeneity of risk aversion in the population (see for instance Binswanger (1980), Jackwerth (2000), and Halek and Eisenhauer (2001)). In particular, Barsky et al. (1997) elicit individual risk tolerance parameters by means of hypothetical questions involving gambles over lifetime income. By polling over 11,000 participants in the Health and Retirement Study, they report an average coefficient of relative risk aversion of about 4, with over 60% of their sample having a coefficient close to 15 and about 10% having a coefficient less than 1. While we do not use their exact implied distribution of coefficients of relative risk aversion, we consider the case where generations may have any integer value of risk aversion between 2 and 10 (inclusive) with uniform probability.

We restrict the university to providing uniform consumption across its members in a given generation and not conditioning its investment policy on any generation's risk aversion. Phrased differently, the university has one policy function at each time over the distribution of risk aversion coefficients, rather than one policy function at each time for each possible level of risk aversion. This is justified by the facts that first, agents are unable to credibly convey their level of risk aversion; second, the university cannot provide different levels of consumption to different members of one generation; and third, the endowment cannot have different asset allocations according to the varying levels of risk aversion within one generation.

Figure 7 presents the optimal investment policies of a university with a fifty-year horizon, fairness levels of 3, 5 and 7 (from top to bottom), and a random risk aversion per generation given by the above uniform distribution. For low levels of wealth (endowment size), the university has little investment in the risky asset (decreasing in fairness), but this risky asset allocation increases as the endowment size rises. Though individual members (generations) of the university may have CRRA utility, the university as a whole behaves as if it has decreasing relative risk aversion. The university prefers the risk free asset at low endowment levels but gradually increases its investment in the risky asset as its endowment size increases.

This ability of our model to accommodate various levels of risk aversion marks an important way in which the preference for fairness is different from risk aversion. Fairness is a higher-order effect taken into account at the university level whereas risk aversion is a feature of the individual members of each generation. Our results show that the high risk-aversion generations matter and force to university to act safely in order to ensure their consumption. This is different from a pure market setting where the low risk aversion agents set prices. However, this effect is overlayed with a wealth effect: As soon as the university's endowment is large enough to guarantee the consumption of the high risk aversion generations, then it gambles much more since the low risk aversion generations' upside benefit is higher than their downside dis-utility.

5.2 Minimum Payout Rule

Endowments and more specifically foundations have mandated minimum payout rules (typically 5%) in order to maintain their tax-exempt status.[10] If the university is constrained to consume at least 5% of its wealth each period, what is the impact on its optimal asset allocation? In untabulated results with a minimum payout rule of 3% (defined as the ratio of consumption over wealth in each period), we show that, for all levels of fairness, the university invests more in the risky than it otherwise would (the benchmark case being the results of Figure 2). Moreover, as the investment horizon lengthens, the optimal risky allocation monotonically rises to 100%. This implies that, no matter how fair the university aims to be, a minimum payout rule pushes the endowment to take on the maximum amount of risk possible. The payout rule is unsustainable and therefore, the university gambles in an attempt to sustain it as long as possible.[11]

It appears as though this result is consistent with the observed behavior of endowments since regents (or the government) mandate high minimum payout rules, which then pushes the asset allocation towards the risky asset. However, a minimum payout rule is not the same as dictating a desired expected return in a mean-variance framework and then picking the efficient portfolio consistent with that return. While this constraint rationalizes the high investment in risky assets *conditional* on the choice of a high payout rule, it leaves unanswered why trustees would choose such a high payout rule voluntarily, an issue that we address in Gilbert and Hrdlicka (2011).

[10] Most universities (although not all) are exempt from this rule.

[11] It is worth noting that universities have multiple sources of income, such as tuition and government appropriations, which can be expected to grow with the overall economy. In this case, intergenerational equity would require a high *current* spending rate out of the endowment.

5.3 Inflation Hedging

The current model has two correlated shocks per period: one shock to expected returns and one shock to realized returns. In untabulated analysis, we include a third shock due to inflation (CPI). One of the endowments' primary goal is to maintain the purchasing power of endowed gifts, which would imply an inflation hedging component to the asset allocation. It is often said that inflation represents a serious risk, then one should increase the allocation to the risky asset. However, this is only true if the inflation shock is correlated with the risky asset's shock. We indeed show that if the two shocks are independent, then the allocation to the risky asset is unchanged in the presence of inflation.[12]

5.4 Left Tail Risk

Our model currently shows that a very fair university will hold little in the way of risky investments. This differs from the observed investment strategy of most endowments. In an attempt to match the observed reality, we consider a university that wishes to be fair to all member of the university but is willing to overlook the possibility of a few extremely bad but low probability events. Such a university is one that does not feel the need to be fair to members of a university who never have the chance to become members because bad investment outcomes essentially put the university out of business.

We implement this willing ignorance as truncation of the left tail of risky asset outcomes. The left tail is truncated both in a given time period and across subsequent time periods. This truncation means that the university ignores the possibility of both terrible returns in a given year and the possibility of a very long string of moderately bad returns, e.g. 50 years of 5 percent losses. This modification should push the university towards holding riskier positions in its endowment. We seek to know the level of left tail risk that needs to be ignored in order to match observed investment patterns.

[12] Note that the universities' inflation, as measured by the Higher Education Price Index (HEPI), is on average 1-2% higher than CPI-measured inflation and its correlation structure with the market also differs.

6 A Model of Equity-Less Producers

In a companion paper Gilbert and Hrdlicka (2011), we model the trustees' mandate of investing an endowment that must fund internal projects, such as research and teaching. Under this mandate, trustees maximizing future cash flows face the trade-off between investing externally or internally. External investment occurs via endowment funds in the capital markets. Internal investment consists of funding current research and teaching, which will generate knowledge and future donations. While facing this trade-off, trustees must simultaneously balance the demands of a diverse set of constituents: internal stakeholders such as faculty and students versus external stakeholders such as donors, alumni, and society at large.

Viewing the university as producer of social goods worthy of donations now to support expanded production is also inconsistent with the observed risky investment policy. On the one hand, the university may not invest more internally in its productive projects because it is financially constrained – the endowment only being able to support current projects and new productive projects being put on hold until new donations arrive. Such constraint has the direct implication that the endowment should be used predominantly as a means of precautionary saving or safety net (buffer) against declines in revenue. But then why are the endowment assets mainly invested in the risky asset? On the other hand, the university may not invest more internally because it has no good projects to invest in. But then why do donors keep donating today when they could delay their donation to tomorrow? With this in mind, if the university has limited (or fixed) risk-bearing capacity, then our research question can be re-phrased: Why does the university take its marginal unit of risk externally (via the capital markets) rather than internally (via new projects)?

This alternative model of universities as *equity-less producers* takes a different stand on the university's objective function. The university produces social dividends through research and teaching. It must finance this work from both its endowment and future donations generated by successful research and teaching. More precisely the university takes risk and generates cash flows in two ways: investing in internal projects that yield tuition and donations; or by investing the endowment in risky capital markets, thereby effectively taking on external projects. The university's optimization is a balancing act between the preferences of donors and internal stakeholders. The model has three key features.

First, universities exist to produce social dividends, which are non-excludable public goods. All members of society consume the social dividends, such as a cure for cancer, but the social dividends cannot be monetized by capital markets. This inability to monetize the result of their production is what makes universities distinct from standard firms. Wealthy former students value the social dividends enough to donate to universities.

Second, due to the non-profit nature of universities and the absence of explicit residual claimants, faculty and administrators have control rights but cannot liquidate the university's assets. Tenure, which provides downside protection, combined with the ability to consume perks, effectively gives the internal stakeholders a convex claim on the university's future cash flows coming from tuition, donations, and endowment. This convex payoff incentivizes the university to willingly take risk to maximize cash flows.

Third, donations from internal projects come far in the future, because alumni take time to accumulate their wealth to the level where they make substantial donations. Though far in the future, these donations represent the majority of the present value of the cash flows generated by new internal projects. In contrast, the faculty and administrators' horizon is far shorter than the duration of these cash flows, because they only retain their claim while employed. Because of this horizon mismatch, for a given amount of risk, universities prefer taking on external projects in capital markets where returns are realized immediately.

Preliminary results show that universities time the market by investing in the outside capital markets when the returns there are better than those on the university's internal projects. These results confirm the intuition that heavy investment in risky assets is a negative signal about a university's projects and that donors should be wary of such a university's ability to effectively use further donations immediately.

It appears as though the demand of internal stakeholders for quick rewards for risk forces the university to forego internal projects with equivalent, or higher, rates of return than the market. Rather, it invests in the capital markets even when some internal projects are marginally better than external projects, and thereby create a welfare loss due to the un-produced social dividends. In such a situation further donations do lead to productive internal investment, but only at a rate lower than one dollar invested for one dollar donated. The donors' problem then becomes distinguishing the amount of investment in the risky asset that marks the boundary between the presence of good or bad internal projects.

7 Conclusion

In this paper, we attempt to explain why university endowment funds have the propensity to hold a large fraction of their assets in risky securities. By taking a stand on the university's objective function, we analyze this question from the oft-cited paradigm that university policies should aim to be intergenerationally fair. We model the university as a consumer who *fairly* maximizes the expected utility over consumption of all future generations. In doing so, we are the first to quantify fairness as a concave meta-utility over each generation's utility. In equilibrium, the fairer the university, the less it invests in the risky asset. In the limit, it completely foregoes the risky asset so as to provide constant consumption.

Under the assumption that universities aim to be fair, the results from our model do not match the observed empirical facts. Two conclusions arise from this analysis: either universities are unfair in their endowment asset allocation decisions or the fair objective function we use is not the one maximized by universities. Taking into account the potential random nature of risk aversion across generations and/or the truncation of extreme left-tail return events can lessen the contradiction. Consistent with Hansmann (1990), we demonstrate that the objective of maintaining fairness does not appear as the main reason for having an endowment in the first place and certainly for managing it in a very aggressive manner.

As Becker and Tomes (1979) highlight, parents may have an inherent preference for fairness inter-generationally (from them to their children) and intra-generationally (between their children) which leads to interesting wealth dynamics. We show that fairness has an impact on risk bearing capacity of universities. This implication of fairness when applied to individuals may lead to further understanding their asset holdings, if fairness and the impact of such holdings on the consumption of their family dynasties matters to their decisions. In a sense fairness may be one more way to explain the implied high risk aversion in the equity premium puzzle.

Our framework for the analysis of the impact of intergenerational fairness on optimal asset allocation can also be used for nonprofit institutions beyond universities. Foundations have the goal of providing a service (grants, research, housing, etc.) over an extremely long period of time and in a way that is fair to all generations. Anecdotally, the tendency to invest in risky assets appears to be widespread among nonprofits. Take for example the Greater Chicago Food Depository, whose liquid endowment assets as of June 30, 2010 were 50% invested in risky assets. Beyond nonprofits, the implications of the preferences for fairness should have implications for the investment policy of public pensions, but we leave that for future work.

A Appendix: Predictability in Expected Returns

In this appendix, we summarize the model of expected returns of Campbell and Viceira (1999), which is further analyzed in Campbell and Viceira (2002) and Campbell, Chan, and Viceira (2003). The log return of the risky asset is defined as

$$r_{t+1} - E_t\left[r_{t+1}\right] = u_{t+1} \tag{8}$$

where u_{t+1} is the innovation to the return and is normally distributed with mean zero and variance σ_u^2. The expected excess stock return is a state variable (x_t) and is defined as

$$E_t\left[r_{t+1}\right] - r_f + \frac{\sigma_u^2}{2} = x_t \tag{9}$$

The state variable is modeled as an AR(1) process with mean μ and persistence ϕ

$$x_{t+1} = \mu + \phi(x_t - \mu) + \eta_{t+1} \tag{10}$$

where the innovation η_{t+1} is normally distributed with mean zero and variance σ_η^2.

The model's key assumption is the covariance between the two innovations, η_{t+1} and u_{t+1}, which is labeled as $\sigma_{\eta\mu}$. This covariance generates intertemporal hedging and x_t represents the investment opportunity set at time t. Expected returns mean-revert when $\sigma_{\eta\mu} < 0$: high returns today are followed by low expected returns tomorrow.

Empirically, the state variable x_t is taken to be the log dividend-price ratio $(d_t - p_t)$, which is known to be a good predictor of stock returns. Using post-war quarterly U.S. financial data, Campbell and Viceira (1999) estimate the following restricted VAR(1) model

$$\begin{pmatrix} r_{1,t+1} - r_f \\ d_{t+1} - p_{t+1} \end{pmatrix} = \begin{pmatrix} \theta_0 \\ \beta_0 \end{pmatrix} + \begin{pmatrix} \theta_1 \\ \beta_1 \end{pmatrix} (d_t - p_t) + \begin{pmatrix} \varepsilon_{1,t+1} \\ \varepsilon_{1,t+1} \end{pmatrix} \tag{11}$$

where the innovations are normally distributed with mean zero and covariance matrix Ω. From the estimated coefficients of the VAR(1) model, the coefficients in equations (8), (9) and (10) can be recovered.

In our calibration, we use the same assumptions and estimates as in the erratum to Campbell and Viceira (1999).[13] The unconditional expected log excess return μ is estimated to be 4.02% per year and the log real risk-free rate r_f is 0.328% per year. The persistence parameter ϕ of the state variable process is 0.957 and the correlation between the η and u innovations is -0.960. The annual volatility of the stock return σ_u is 15.253% and the annual volatility of the state variable σ_η is 1.166%. Finally, the rate of time preference δ

[13]The erratum is available on Campbell's website.

is set to 0.92 in annual terms.

B Appendix: Numerical Solution Methodology

The university has an infinite horizon and maximizes its expected fairness utility over each generation's consumption:

$$\max_{c_t, \alpha_t} E_t \lim_{T \to \infty} \sum_{i=0}^{T} \frac{1}{T} V\left[u(c_{t+i})\right] \tag{12}$$

where the utility functions are given by

$$u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma} \qquad \text{and} \qquad V\left[u(c_t)\right] = \frac{\left(-\frac{1}{u(c_t)}\right)^{1-f}}{1-f}. \tag{13}$$

The intertemporal budget constraint is

$$\tilde{W}_{t+1} = (W_t - c_t) \times (\alpha_t \tilde{R}_{r,t+1} + (1-\alpha_t) R_f). \tag{14}$$

Due to the model's analytical non-tractability, we solve it numerically by state-space discretization.

We approximate a normal distribution for the expected and realized return shocks by using the Gauss-Hermite quadrature method. We use three states and the shocks are correlated according to formulas on two-dimensional Gauss-Hermite quadrature available in Jäckel (2005).

There are two state variables, wealth and expected returns, and the optimization grid has about 975 points: 75 along the wealth dimension (1 to 75 in intervals of 1) and 13 along the returns dimension (from minus 3 to plus three standard deviations in intervals of one-half the standard deviation).

At each iteration step, the value function is inverted to its certainty equivalent. Following Garlappi and Skoulakis (2009), this step removes most of the highly nonlinear features of CRRA utility, especially at high levels of risk aversion. As a result, the interpolation of the value function, a necessary step that is done at each iteration, is much faster and robust.

The numerical maximization method used is that of Nelder-Mead, which is a version of the simplex method for non-linear programming.

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Figure 1
Indifference Curves of Fairness Utility

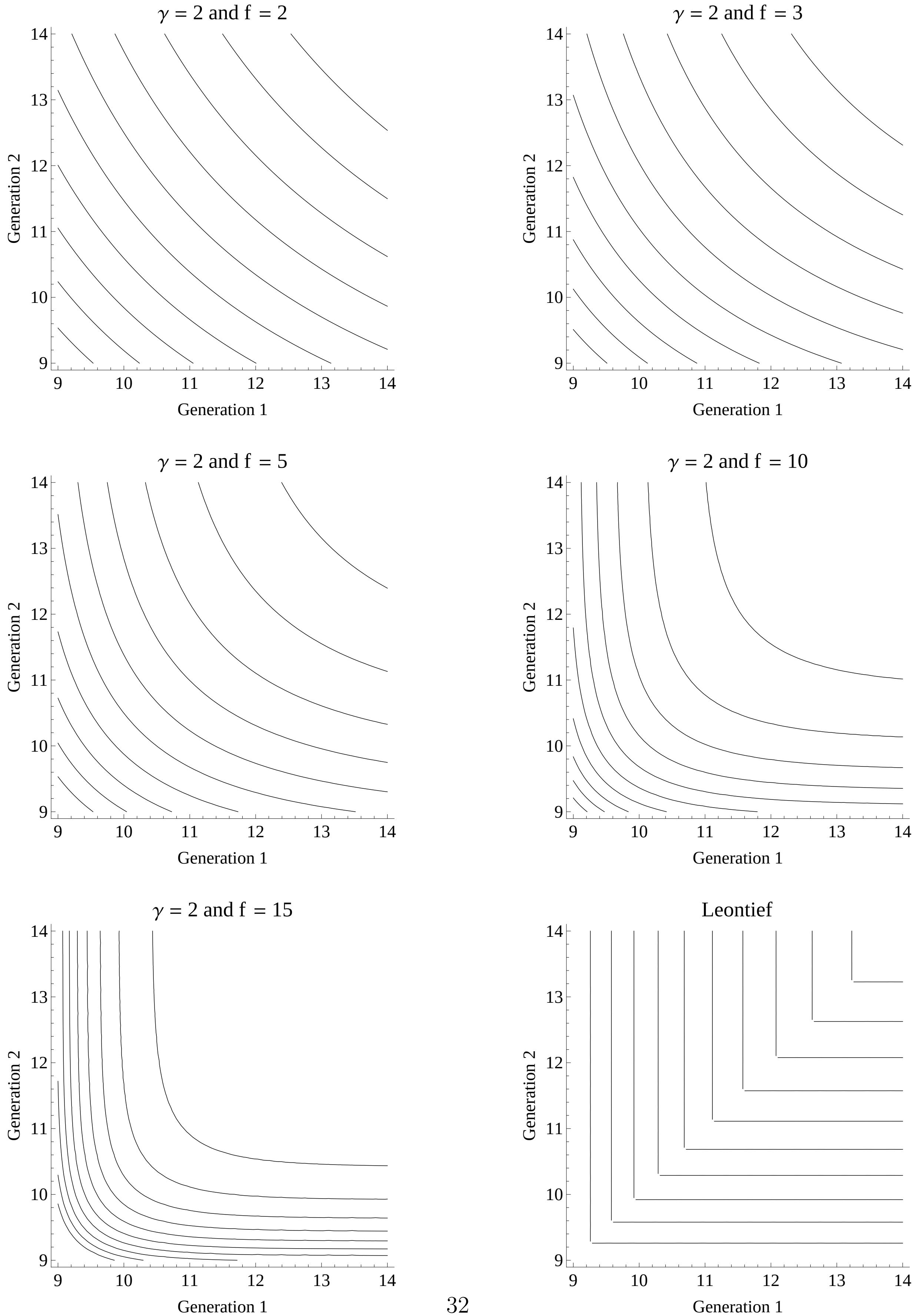

Figure 2
Risky Asset Weight vs. Investment Horizon

In this figure, we plot the optimal asset allocation to the risky asset as a function of the university's horizon. All models are run with a coefficient of relative risk aversion of 5. The dotted line represents the model with i.i.d. returns and standard CRRA utility. The top line is the model with non i.i.d. returns and standard CRRA utility. The other lines (from top to bottom) are models with non i.i.d. returns and increasing fairness coefficients, from 3 to 10. In the second panel, the investment horizon is increased to 1,000 years, returns are non i.i.d. and fairness is increased from 2 to 10.

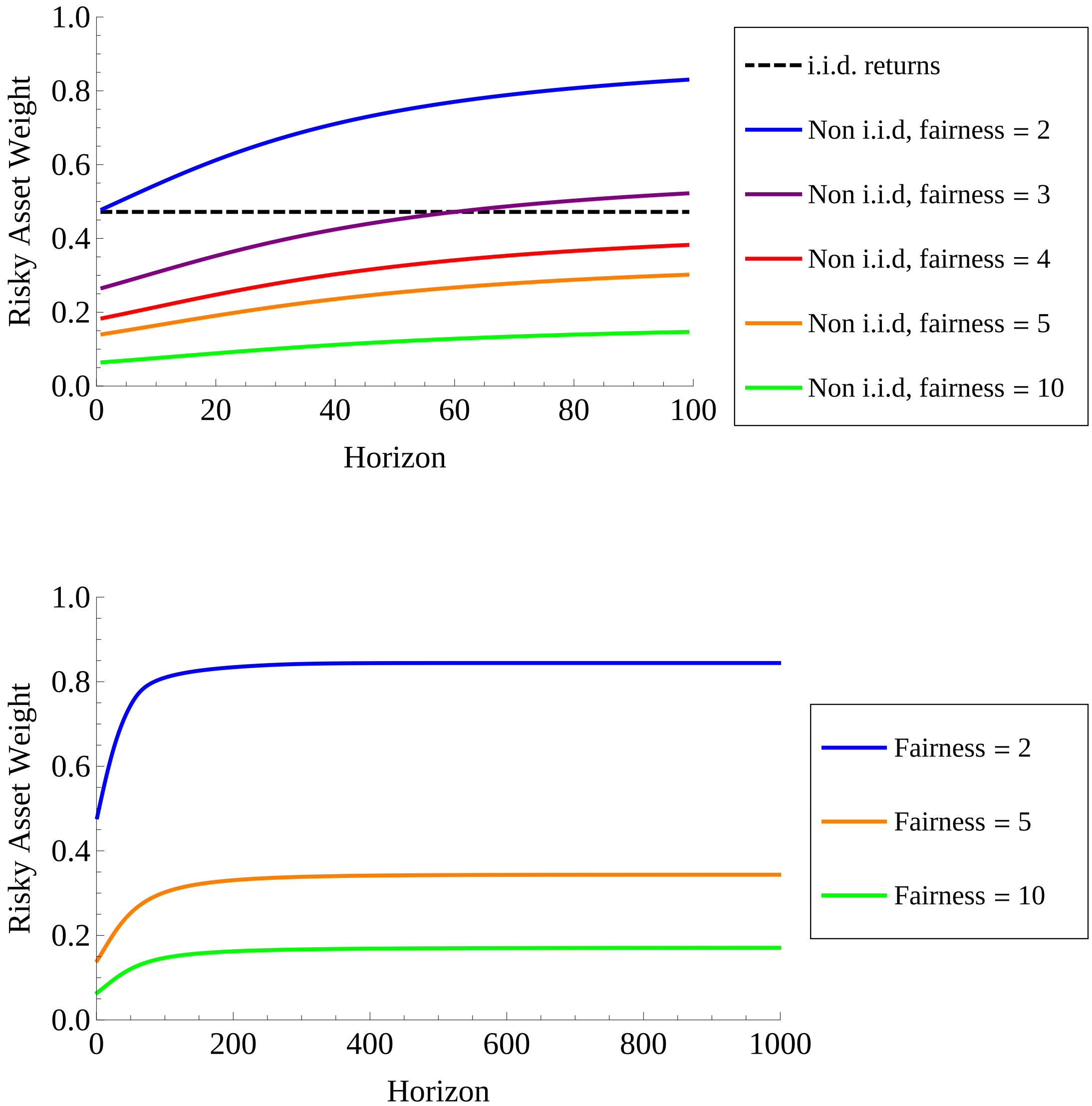

Figure 3
Risky Asset Weight vs. Fairness

In this figure, we plot the optimal asset allocation to the risky asset as a function of the university's fairness coefficient. All models are run with a coefficient of relative risk aversion of 5. The dotted line represents the model with a one-period horizon (i.i.d. returns). From bottom to top, the lines represent an increase in the investment horizon, from 25 to 100 years.

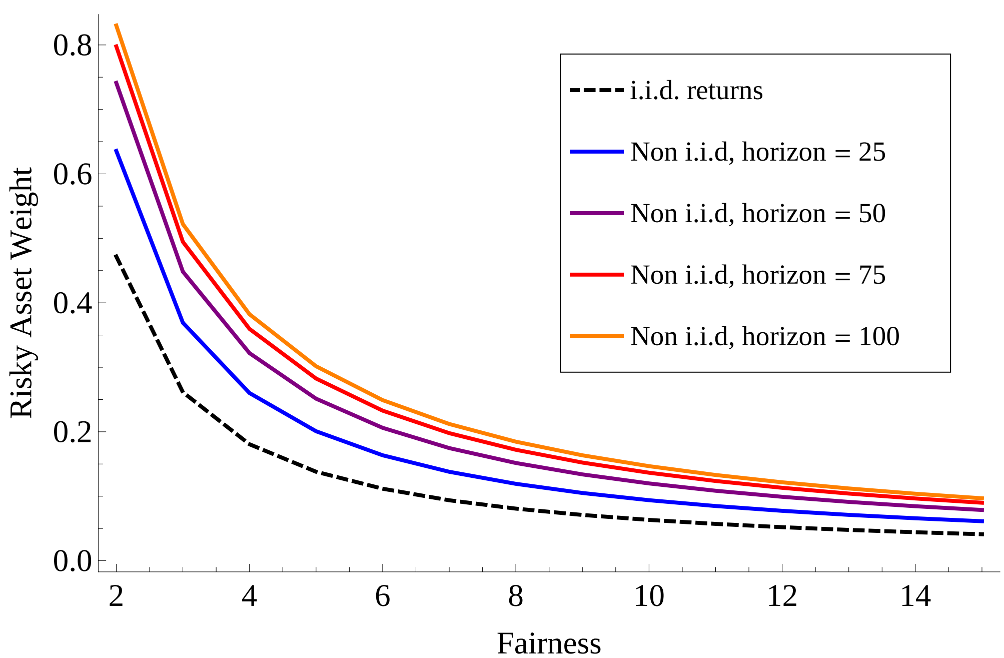

Figure 4
Risky Asset Weight vs. Expected Returns

In this figure, we plot the optimal asset allocation to the risky asset as a function of the risky asset's expected return. All models are run with a coefficient of relative risk aversion of 5. The dotted line represents the model with i.i.d. returns. The top line is the strategic asset allocation for a long-term investor who times the market in the face of non-i.i.d. mean-reverting returns. The middle line is the tactical asset allocation of the single-period investor who buys and holds. The bottom line is the optimal allocation for a fair university with non-i.i.d. returns and long-horizon.

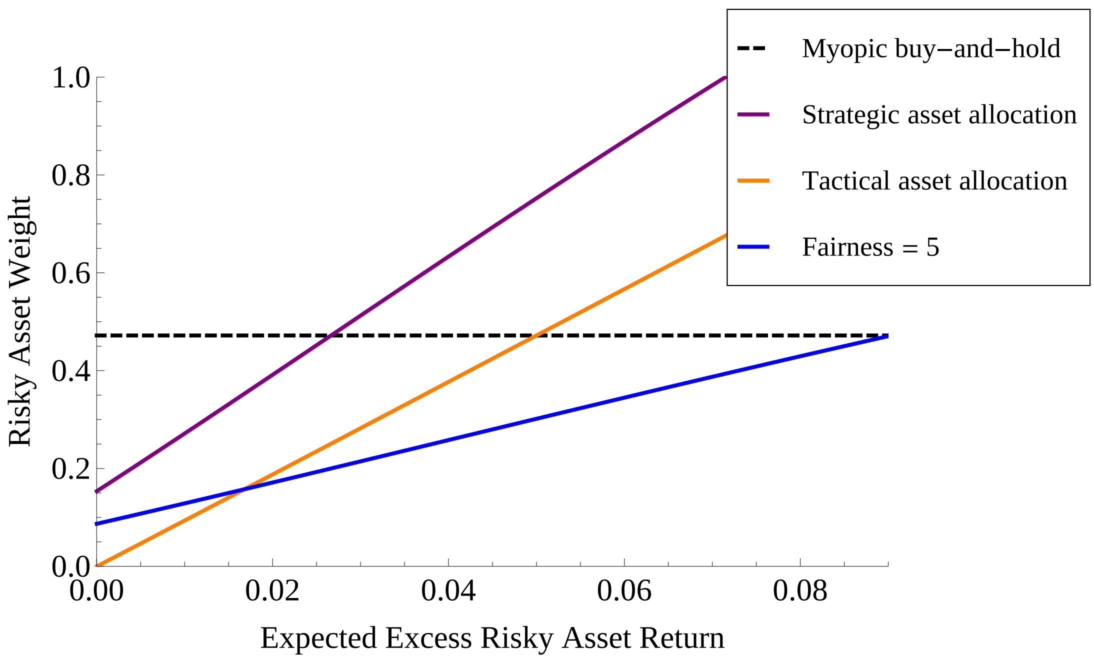

Figure 5
Impact of Zero Rate of Time Preference

In this figure, we plot the optimal asset allocation to the risky asset as a function of the university's horizon. All models are run with a coefficient of relative risk aversion of 5 and a fairness coefficient of 10. The top line is the model with a zero rate of time preference ($\delta = 1$) and an equal-weighting of all generations, whereas the bottom line is the model with the standard time discounting $\delta < 1$.

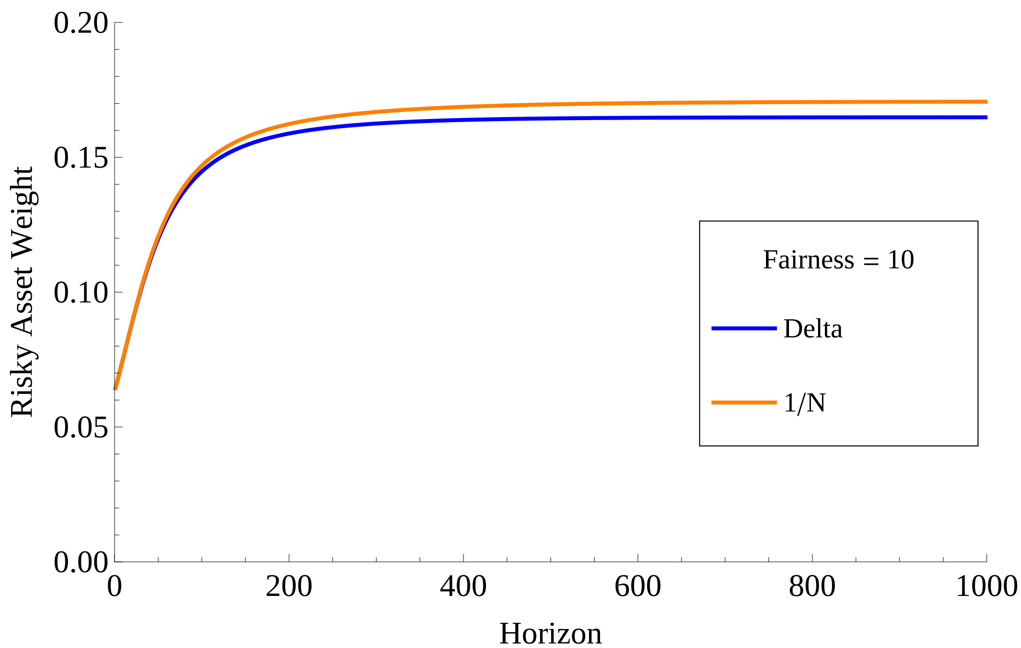

Figure 6
Consumption Volatility vs. Investment Horizon

In this figure, we plot the volatility of consumption as a function of the university's investment horizon. All models are run with a coefficient of relative risk aversion of 5. From top to bottom, the lines represent an increase in the coefficient of fairness: 2, 3, 5, 10, and 15. Consumption paths are obtained by simulating 1,000 return paths and running the optimal consumption and investment policies through each time series of returns.

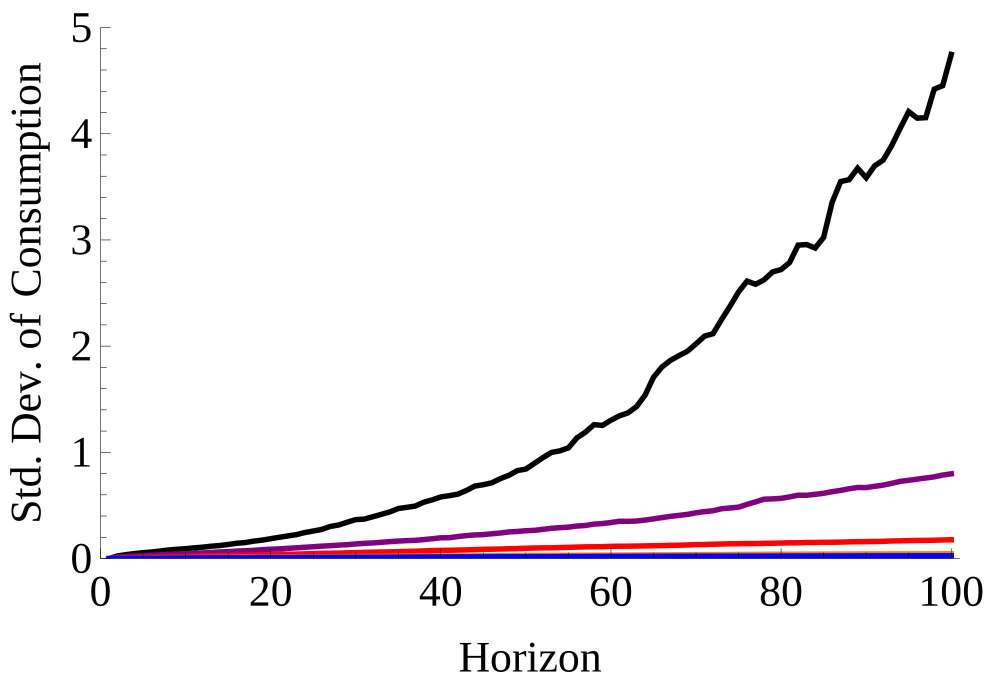

Figure 7
The Impact of Random Risk Aversion on Optimal Asset Allocation

In this figure, we plot the optimal allocation to the risky asset as a function of wealth for different levels of fairness (2, 5, and 7 from top to bottom) when each generation is randomly assigned a coefficient of relative risk aversion between 2 and 10 with uniform probability.

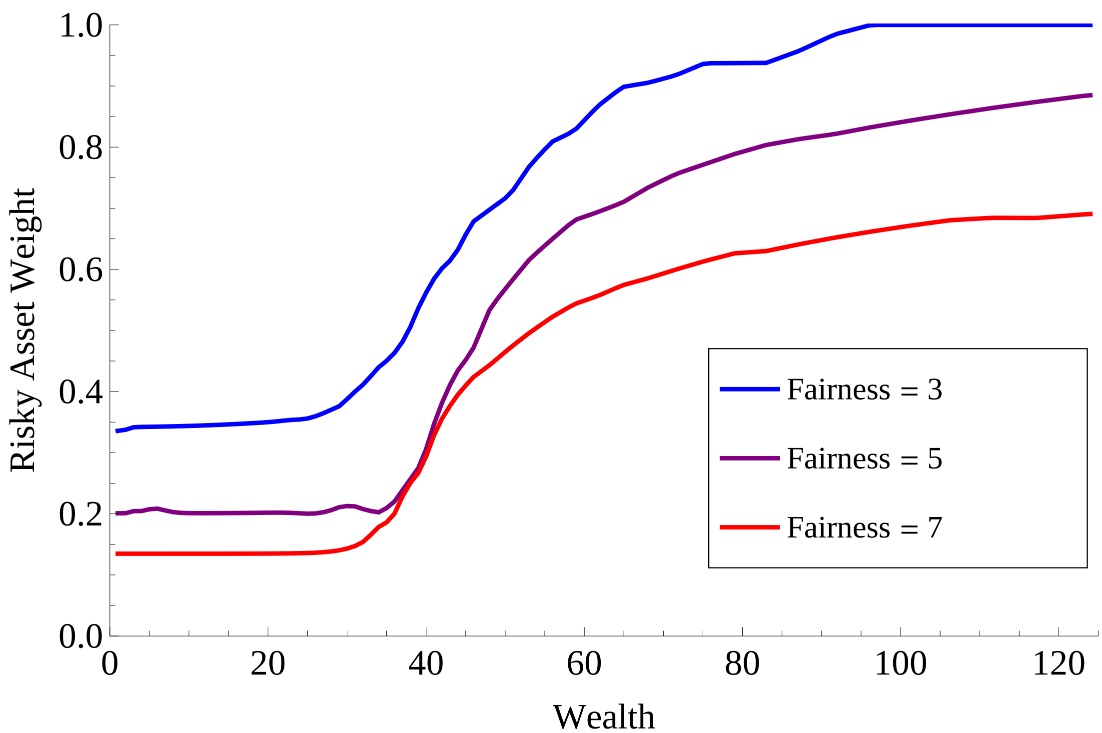

Table I
Consumption Dynamics

This table presents measures of consumption behavior across levels of fairness at the steady-state solution of our model (1,000-year horizon). The payout ratio is defined as consumption divided by endowment size. The expected portfolio return is defined as the endowment's return given by the optimal asset allocation policy. IRF stands for Impulse Response Function and measures the percentage change in consumption for a one standard deviation shock in returns. Equal-weighting is the model with a zero rate of time preference and δ is the model with discounting.

	Equal-Weighting				δ
Fairness Coefficient	2	5	10	15	10
Payout Ratio	2.04%	0.99%	0.62%	0.51%	0.85%
Expected Portfolio Return	4.37%	1.95%	1.11%	0.83%	1.08%
Payout Ratio / Expected Portfolio Return	47.51%	51.09%	56.11%	60.96%	79.50%
IRF: Percentage Change in Consumption	13.31%	5.52%	2.76%	1.84%	2.67%